Computing the expected signal per pixel using TELESTO telescope

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1 Introduction

Determining the optimal signal and exposure time is essential in astronomical observations to achieve a sufficient signal-to-noise ratio (SNR) while avoiding detector saturation.
This document explains how to calculate the expected signal for observing an astronomical object using the Telesto telescope.

Section 2 presents the complete formula used to compute the signal in analog-to-digital units (ADU) as measured by the detector.
Section 3 details the derivation of this formula, outlining the physical principles and unit conversions involved.
Section 4 provides two worked example demonstrating how to apply the formula numerically.

A corresponding Python Jupyter notebook tutorial is included on the Telesto webpage `https://plone.unige.ch/astrodome/observations/content/jupyter-notebook-tutorial/view`, offering a practical implementation of the method for computational signal estimation.

As a brief introduction, we provide explanations of a few fundamental concepts in astronomical detection and instrumentation below. Feel free to skip this part if you're already familiar with them.

1. **Exposure time** is how long a telescope looks at a star or galaxy. Just like your eyes see more in the dark if you stare longer, a telescope collects more light from an object the longer the exposure time t. More exposure time means a brighter and clearer image.

2. **Saturation** happens when a detector receives too much light and cannot count any more. It's like pouring too much water into a full glass—it spills over. In detectors, this means the pixel shows its maximum value and can't measure anything higher.

3. **Gain** tells us how many electrons equal one digital number on the image (called ADU, see below). If the gain is 1, then 1 electron = 1 ADU. If the gain is 2, then it takes 2 electrons to get 1 ADU. It's like saying how many coins you need to buy a candy.

4. **Quantum efficiency (QE)** tells us how good a detector is at turning light (photons) into electric signal (electrons). If a detector has a QE of 0.8 (or 80%), then 8 out of 10 photons are counted. The rest are lost. Higher QE means better sensitivity.

5. **Signal-to-Noise-Ratio (SNR)** tells us how clear a signal is. It compares the real light from the object (signal) to the random bumps or fuzziness (noise). A higher SNR means a clearer image. Common SNR values:

   - SNR > 100: very clear signal (used in precise measurements)
   - SNR ~ 30: good for most images
   - SNR $\sim 5 - 10$: faint object, barely visible

6. **Analog-to-Digital-Unit (ADU)** It is the number saved in the image for each pixel. It tells us how much light was received. Bigger ADU = more light. The ADU depends on the number of photons, the QE, the gain, and the exposure time.

7. **Point Spread Function (PSF)** is how a single point of light (like a star) looks on the detector. Because of the telescope optics, even a tiny dot of light is spread over several pixels. The PSF shape depends on the telescope, the atmosphere, and the detector. It usually looks like a blurry spot with a bright center.

8. **Transmission function** tells us how much light goes through something—like a filter or a telescope lens—at each color (wavelength).

   Imagine shining a rainbow through a piece of colored glass. Some colors go through easily, while others get blocked. The transmission function is like a scorecard that says:

   - Blue light: 80% goes through
   - Green light: 60% goes through
   - Red light: 20% goes through

   So the transmission function depends on the wavelength (the color). It usually looks like a curve that shows how well each color passes through a filter or instrument.

9. **Apparent magnitude** tells us how bright an object appears in the sky from Earth. Smaller numbers mean brighter objects. Magnitudes are always measured through a specific filter or bandpass, which means they represent the brightness averaged over a certain range of wavelengths (colors). When stating an apparent magnitude, you should always specify the bandpass it refers to (such as visible, infrared, etc.). For example, in the visible:

   - The Sun: magnitude -26
   - Full Moon: magnitude -12
   - Bright star (like Sirius): magnitude -1.4
   - Naked eye limit: about magnitude $+6$

   A change of 5 magnitudes means 100 times difference in brightness!

10. **Spectral Energy Distribution (SED)** shows how much light the star gives off at each color (wavelength). It comes from the star's surface and depends mostly on its temperature. usually it's a graph where the x-axis is the wavelength (color of light), and the y-axis is how bright the star is at that color.

    Hot stars shine more in blue light. Cool stars shine more in red light. Even if the star looks white to us, its SED reveals its true energy at every color.

    Different types of stars and galaxies have different-shaped SEDs, just like people have different voices. By looking at the shape of the SED, astronomers can tell what kind of object it is, how hot it is, how far away it is, and more!

2 Signal computation

The formula below provides the complete expression used to calculate the signal in ADU as recorded by the detector. It can be used to estimate the optimal exposure time required to reach a target signal level. Since the Telesto detector saturates at approximately 65,500 ADU (32-bit system), it is recommended to keep the signal well below this limit—ideally in the range of 30,000 to 40,000 ADU—to maintain detector linearity and avoid saturation.
Given an astronomical source, the signal in ADU recorded by the detector can be calculated using the following formula:

$$S = \frac{1}{N_{\text{pixels}}} \frac{1}{\text{Gain}} \frac{1}{hc} \cdot A_{\text{tel}} \cdot T_{\text{tel}} \cdot t \int_{\lambda} F_{\lambda,\text{Earth}} \cdot T_{\lambda} \cdot QE_{\lambda} \cdot \lambda \, \mathrm{d}\lambda \quad \text{[ADU]} \tag{1}$$

For a complete derivation of this formula, please refer to the next section.
The terms in the equation are defined as follows:

- $N_{\mathbf{pixels}}$: Number of detector pixels over which the astronomical object is distributed. For point sources such as stars, the light typically spreads over multiple pixels due to the instrument's point spread function (PSF), which results from the telescope's optical response. In the case of the Telesto telescope, the PSF typically covers a region of approximately 4×4 pixels, corresponding to a total of 16 pixels.

  For extended sources like galaxies or nebulae, this value depends on the object's angular size on the sky and the instrument's plate scale. To estimate it accurately, see the Telesto website page: `https://plone.unige.ch/astrodome/observations/content/how-to-compute-the-target-size-in-pixel`

- **Gain**: Conversion factor between the number of photoelectrons detected and the corresponding digital value recorded in ADU (Analog-to-Digital Units). For the Telesto camera (QHY600M Pro[1]), the gain typically ranges from 0.4 (to maximize sensitivity for faint sources) to 1.6 (to preserve dynamic range for bright sources). A value of 1 is commonly used as a balanced setting in most observations.

- h: Planck's constant, $h = 6.62607015 \times 10^{-34}$ J·s.

- c: Speed of light in a vacuum, $c = 2.99792458 \times 10^{8}$ m/s.

- $A_{\mathbf{tel}}$: Effective collecting area of the telescope. Telesto has a primary mirror of diameter $D = 30$ cm. However, due to the central obstruction (the spider structure + secondary mirror), approximately $O = 17.2$ cm in diameter is blocked. The effective collecting area is calculated as:

$$A_{\text{tel}} = \pi \left[\left(\frac{D}{2} \right)^2 - \left(\frac{O}{2} \right)^2 \right] \approx 0.190\,\text{m}^2$$

- $T_{\mathbf{tel}}$: Overall transmission efficiency of the telescope's optics, accounting for losses from mirror reflectivity, coatings, and alignment. While the true throughput is wavelength-dependent, we assume a constant average value of 70%, based on the typical performance of similar optical systems.

- t: Exposure time, expressed in seconds.

All quantities inside the integral are wavelength-dependent. Sections 4 present two approaches for evaluating the integral. The first is a faster, approximate method based on the object's apparent magnitude. The second is more comprehensive, relying on the full spectral energy distribution of the source.
For clarity, the wavelength-dependent terms in the integral are defined below:

- $F_{\lambda,\mathbf{Earth}}$: Flux density of the astronomical object as measured at Earth, expressed in $\text{erg}\,\text{s}^{-1}\,\text{m}^{-2}\,\text{Å}^{-1}$. Section 3.1 details how this quantity can be estimated either from the object's apparent magnitude or from its full spectrum.

- T_λ: Transmission curve of the selected photometric filter. A list of available filters and their transmission profiles is available on the Telesto website: `https://plone.unige.ch/astrodome/characteristics-of-telesto`

- QE_λ: Quantum efficiency of the detector, i.e., the fraction of incoming photons that are converted into photoelectrons. This efficiency is wavelength-dependent. The QE curve for the Telesto detector is available on the camera manufacturer's website: `https://www.qhyccd.com/astronomical-camera-qhy600/` or via the Telesto technical documentation: `https://plone.unige.ch/astrodome/characteristics-of-telesto`

- λ: Wavelength, expressed in Angstroms (Å).

[1] `https://www.qhyccd.com/astronomical-camera-qhy600/`

3 Formula derivation

3.1 Flux density estimation

The central quantity in Eq. 1 is the flux density of the astronomical object as observed from Earth, denoted $F_{\lambda,\text{Earth}}$. This can be estimated using two different approaches: (1) from the object's apparent magnitude, or (2) from its full spectral energy distribution.

1. Flux density from apparent magnitude

In photometric systems, $F_{\lambda,\text{Earth}}$ is commonly derived from the star's apparent magnitude using a known zero-point flux, usually based on the star Vega. Although Vega is often used as the standard for magnitude zero in all filters, this is only an approximation. In reality, Vega's magnitude varies slightly between bands due to its spectrum and the finite width of each filter's transmission curve.

The relation between magnitudes m_1 and m_2, and their corresponding fluxes F_1 and F_2, is given by:

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_{1,\text{Earth}}}{F_{2,\text{Earth}}}\right) \tag{2}$$

Solving for $F_{1,\text{Earth}}$, we obtain:

$$F_{1,\text{Earth}} = F_{2,\text{Earth}} \cdot 10^{-0.4(m_1 - m_2)} \tag{3}$$

In the special case where $m_2 = 0$, we can write:

$$F_{1,\text{Earth}} = F_0 \cdot 10^{-0.4 m_1} \tag{4}$$

As already stated in the introduction, note that the apparent magnitude refers to a specific photometric filter (visible, infrared, etc.). Since filters have finite bandwidth, the resulting flux is not strictly monochromatic but rather a weighted average across the filter's transmission profile. This can be denoted as an average quantity: $\overline{F_{\lambda,\text{Earth}}} = F_{1,\text{Earth}}$.

For some values of the zero-point F_0, please refer to table1.[2]

2. Flux density from stellar spectrum

A more precise approach uses the star's intrinsic spectrum, F_λ, which is defined at the surface of the star in units of $\text{erg}\,\text{s}^{-1}\,\text{m}^{-2}\,\text{Å}^{-1}$. This quantity describes the energy emitted per unit wavelength, area, and time. The total luminosity per wavelength from the star is (spherical emitting surface):

$$F_{\lambda,\text{total}} = F_\lambda \cdot 4\pi r_\star^2$$

At Earth, located a distance $d_\star$ from the star, the flux density becomes:

$$F_{\lambda,\text{Earth}} = F_\lambda \cdot \left(\frac{r_\star}{d_\star}\right)^2 \quad \left[\text{erg}\,\text{s}^{-1}\,\text{m}^{-2}\,\text{Å}^{-1}\right]$$

3.2 Photon flux and instrument response

Once the flux density at Earth, $F_{\lambda,\text{Earth}}$, is known, it can be converted into a photon flux using the energy of a photon:

$$E_\gamma = \frac{hc}{\lambda}$$

Therefore, the number of photons per unit area, time, and wavelength is:

$$\Phi_\lambda = \frac{F_{\lambda,\text{Earth}}}{E_\gamma} = F_{\lambda,\text{Earth}} \cdot \frac{\lambda}{hc} \quad [\text{photons}\,\text{s}^{-1}\,\text{m}^{-2}\,\text{Å}^{-1}]$$

[2] A widely used rule-of-thumb is that Vega's flux in the visible band is approximately $F_0 \approx 1000$ photons $\text{cm}^{-2}\,\text{s}^{-1}\,\text{Å}^{-1}$, centered at 550 nm.

The total number of photons entering the telescope per second and per unit wavelength is:

$$N_\lambda = \Phi_\lambda \cdot A_{\rm tel} = F_{\lambda,\rm Earth} \cdot \frac{\lambda}{hc} \cdot A_{\rm tel} \quad [\text{photons}\,\text{s}^{-1}\,\text{Å}^{-1}]$$

This photon rate is then reduced by instrumental effects. The photons first pass through the telescope optics, experiencing loss modeled by the telescope transmission $T_{\rm tel}$. They also pass through a photometric filter with wavelength-dependent transmission T_λ. The signal is spread over multiple pixels (typically 16 for point sources with Telesto), so the result is divided by $N_{\rm pixels}$.
Next, the detector converts photons to electrons with a wavelength-dependent quantum efficiency QE_λ, and finally into ADU via the detector's Gain setting.
Combining all effects, the signal per wavelength becomes:

$$N_\lambda = F_{\lambda,\rm Earth} \cdot \frac{\lambda}{hc} \cdot A_{\rm tel} \cdot T_{\rm tel} \cdot T_\lambda \cdot \frac{1}{N_{\rm pixels}} \cdot QE_\lambda \cdot \frac{1}{\rm Gain} \quad [\text{ADU}\,\text{s}^{-1}\,\text{Å}^{-1}] \tag{5}$$

Since observations are performed over a finite integration time t, we multiply the expression by t to obtain the total signal.
Finally, this expression is monochromatic. To compute the total signal across a finite bandwidth (e.g., a filter), the entire equation must be integrated over wavelength—yielding Eq. 1.

4 Two possible approaches

4.1 Simplified approach using apparent magnitude

This method provides a quick and practical way to estimate the signal described by Eq. 1 without explicitly solving the integral. It relies on the object's apparent magnitude and uses mean values for filter and detector properties.
The necessary parameters—effective wavelength, bandwidth, zero-point flux, mean transmission, and mean quantum efficiency—are provided in the table below. These values are derived partly from the literature and partly from specific measurements or calibrations for the Telesto telescope system.

FILTER	**Effective Wavelength** (Angstroms)	$\Delta\lambda$ (Angstroms)	**Zero-point** F_0 ($\text{erg}\,\text{s}^{-1}\,\text{cm}^{-2}\,\text{Å}^{-1}$)	**Mean Transmission**	**Mean QE (QHY600)**
U	3598.5	697	4.34×10^{-9}	0.70	0.5
B	4385.9	888	6.40×10^{-9}	0.83	0.7
V	5490.6	814	3.67×10^{-9}	0.81	0.82
R	6939.5	2093	1.92×10^{-9}	0.81	0.5
I	8780.8	2206	9.39×10^{-10}	0.85	0.19

Table 1: Effective wavelengths, FWHM (Full Width at Half Maximum) bandwidths, zero-point fluxes F_0, mean transmissions, and QHY600 mean quantum efficiencies for Johnson-Cousins (Bessell) filters.

To simplify Eq. 1 using mean values, we approximate each wavelength-dependent quantity as follows:

- $F_{\lambda,\rm Earth}$: Use Eq. 4 with the zero-point flux F_0 from Table 1, treating the result as an average flux across the filter band.
- T_λ: Use the mean transmission value listed for the filter.
- QE_λ: Use the mean detector quantum efficiency provided.
- λ: Use the effective wavelength of the filter.

The integral can then be approximated as:

$$\int_\lambda F_{\lambda,\text{Earth}} \cdot T_\lambda \cdot QE_\lambda \cdot \lambda \, d\lambda \approx \overline{F_{\lambda,\text{Earth}}} \cdot \overline{T_\lambda} \cdot \overline{QE_\lambda} \cdot \lambda_{\text{eff}} \cdot \Delta\lambda = F_{1,\text{Earth}} \cdot \overline{T_\lambda} \cdot \overline{QE_\lambda} \cdot \lambda_{\text{eff}} \cdot \Delta\lambda \tag{6}$$

This approximation allows a quick estimation of the signal using only tabulated quantities and avoids the need for detailed spectral integration. A Jupyter notebook tutorial demonstrating a possible computational implementation of this calculation is available on the Telesto website: `https://plone.unige.ch/astrodome/observations/content/jupyter-notebook-tutorial/view`.

4.2 More accurate approach: full spectral integration

A more precise alternative involves solving the full integral numerically. This requires:

- The spectral energy distribution (SED) of the astronomical object
- The filter transmission curve T_λ
- The detector quantum efficiency curve QE_λ

These curves are available from the Telesto website and related instrument documentation.
This method accounts for the detailed wavelength dependence of all relevant components and is especially important when observing stars with strong spectral features or using filters with irregular shapes.
In the Telesto webpage `https://plone.unige.ch/astrodome/observations/content/jupyter-notebook-tutorial/view`, we demonstrate how to implement this computation in Python using real data and numerical integration (Jupyter notebook tutorial).